Introduction to Nonlinear Time Series Analysis

Capstone Seminar, Truman State University

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Advised by Dr. David Garth

Outline

What Is Next

History

Nonlinear time series analysis

- Traces back to Henri Poincaré's three-body problem in the late 1800s.
  - Refers to the challenge of accurately predicting the future positions and motions of three celestial bodies.
  - Poincaré laid the foundations of chaos theory.
- In the 1960s, Edward Lorenz formulated a system of equations.
  - Highlighted the sensitivity to initial conditions, a hallmark of chaotic systems.
  - Guided researchers towards nonlinear dynamics.
- Simultaneously, Mandelbrot studied fractals.
  - Geometric shapes containing detailed structures at small scales. They repeat patterns as you zoom in.

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Climate variability

- Natural fluctuations occur over various timescales, from seasons to millennia.
- NTSA gained prominence in climate science during the late 20th century.
  - Researchers sought to study the inherently nonlinear behavior of our planet's climate system.
  - Increase in computational power.
- Currently, there are many NTSA methods used, some are
  - Rescaled range analysis
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  - Singular spectrum analysis
  - etc.

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Important Work

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  - Mentioned earlier– sensitivity to initial conditions
  - Famous term *butterfly effect*; revolutionized meteorology and physics.
- Geometry from a Time Series by Packard et al., 1980
  - Proposed *phase space reconstruction*– we will talk about this soon.
  - This paper is essential for anyone interested in chaos theory, nonlinear dynamics, and the practical applications of these concepts in fields ranging from physics to biology and engineering.

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- Time series
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- Trajectory: the path created when we plot data points; shows us how a system changes.
- Deterministic vs. Nondeterministic systems
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State Space Reconstruction

- Abstract representation of a dynamic system's complete condition.
  - A multidimensional space where each dimension corresponds to a variable that somehow describes the system. Consider how temperature describes the overall weather.
  - A *state* represents a snapshot of the system. Analogous to a data point in a time series.
- But why use this technique?
  - We do not always know all the internal variables!
  - Reconstruction using temperature may be similar to, say, reconstruction using precipitation.

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Delay Coordinate Embedding

- Reconstructs a state space from a single time series.
  - More than one dimension from a scalar time series– how is that possible?
- Consider a scalar measurement x, say temperature. We can construct an m-dimensional vector $\overrightarrow{R}(t)$ from m time-delayed measurements $x(t)$, such that

  $$\overrightarrow{R}(t) = [x(t), x(t-\tau), x(t-2\tau), ..., x(t-(m-1)\tau)]$$

  where t is the time of measurement and τ is the chosen time delay. The time-delay variable τ represents the intervals of these measurements.
- How does this translate to practical work?

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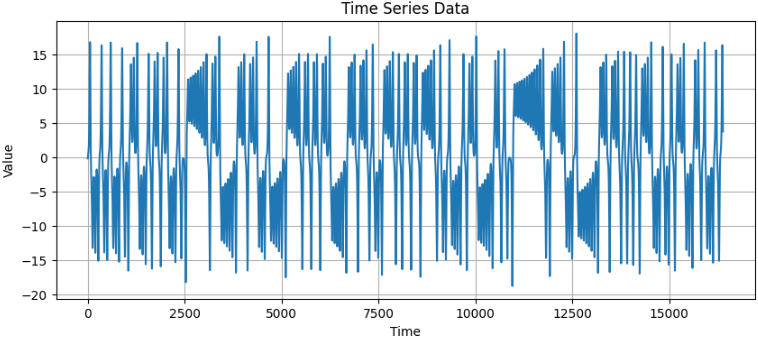
Delay Coordinate Embedding Cont.

Example to visualize

Consider the following time series



Delay Coordinate Embedding Cont.

Example to visualize

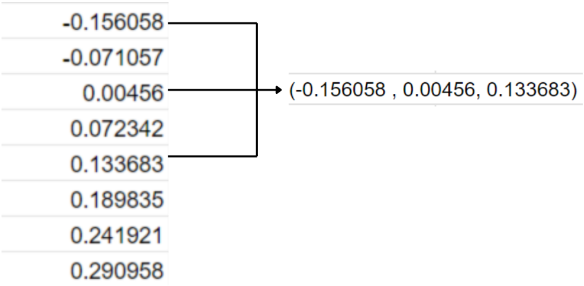
Here are the first eight data points

-0.156058
-0.071057
0.00456
0.072342
0.133683
0.189835
0.241921
0.290958

Delay Coordinate Embedding Cont.

Example to visualize

Let's embed some points!



Notice the time interval τ is two. Choosing τ is also a difficult task that is studied on its own– what value is too little vs. too much?

Delay Coordinate Embedding Cont.

Example to visualize

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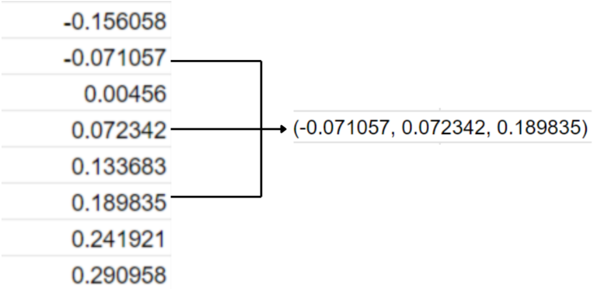

We can plot these points in a 3D space. When we do this for all data points, we can plot the trajectory.

What Is Next

Lyapunov Exponents, λ

- Russian mathematician Aleksandr Lyapunov.
- Quantifies the sensitivity of a dynamical system to its initial conditions.
- There is more than one exponent for a system– one for each variable. However, only one determines the overall behavior.
- $\lambda > 0$, signifies chaotic behavior within the system.
- Common method used: Rosenstein's algorithm

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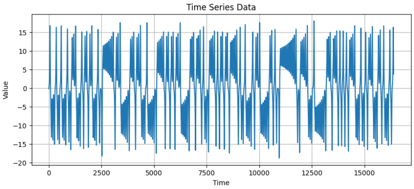

The Lyapunov exponent for this time series is 0.9056.

Lyapunov Exponents, λ

Rosenstein's Algorithm

1. Embedding: Reconstruct the state space using delay-coordinate embedding.
2. Nearest neighbor search: $d_e = ||X_i - X_i||$, where $i - e > \mu$
3. Logarithmic growth: Track how the distance between data points evolves.
4. Estimation: Analyze the rate of this divergence.
5. Finding the exponent: Take the slope of a graph showing the natural logarithm of these distances.

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Attractors

- Mathematical constructs that provide insight into the long-term evolution of dynamic systems.
- Three kinds
  1. Fixed-point attractors represent rest.
  2. Periodic attractors represent repeating patterns or cycles, such as periodic orbits.
  3. Strange attractors represent complex, non-repeating attractors found in chaotic systems. Turbulence is a prime example of such behavior.
- Lorenz first noticed chaotic systems in the behavior of these three equations.

$$dx/dt = -ax + ay$$

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Attractors Cont.

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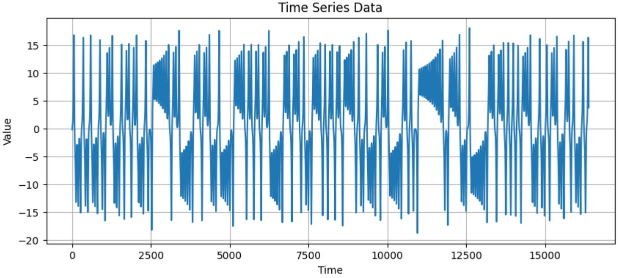
The time series produced by these equations looks like this

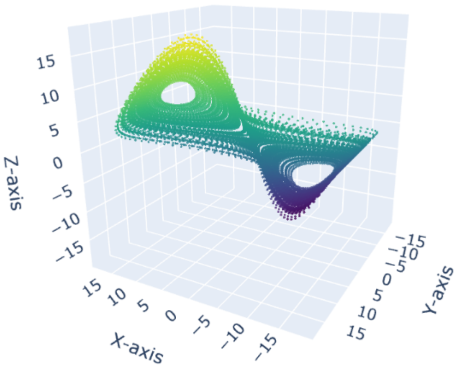


We have been looking at the Lorenz attractor's time series.

What does it look like?

The Lorenz Attractor

Choosing $\tau = 10$, we get



Let's try other τ values to see their effect on reconstruction.

Click here!

What Is Next

Conclusion

Here is what we covered today:

1. Introduction
    - How NTSA came to be
    - Some important work in the field
2. Foundations
    - Basic terminology
    - State space reconstruction
    - Delay coordinate embedding
3. Characterization
    - Lyapunov exponents
    - Attractors

Overall, you should have a basic understanding of how nonlinear time series analysis works.

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Thank you for listening!

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